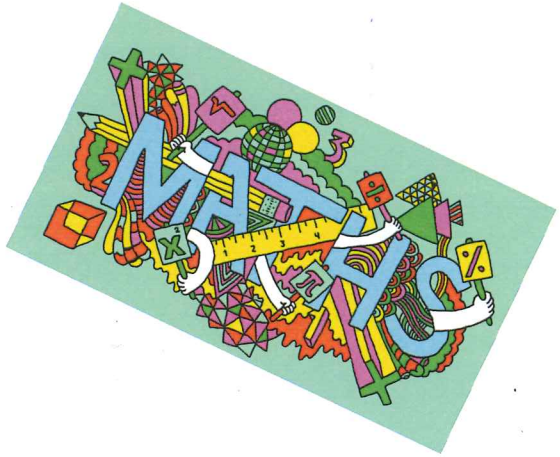
Progression in Calculation

at

St John the Baptist Catholic Primary School



Viam Parabimus

Aims

The national curriculum for mathematics *(2014)*

aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. ...pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to Science and other subjects.

National Curriculum 2014

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the children can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year R: U + U, U - U, doubling single digit numbers, halving and sharing numbers up to 10

Year 1: U + U, U + TU (numbers up to 20) including adding zero, U - U, TU - U (numbers up to 20) including subtracting zero, U x U, U ÷ U

Year 2: TU + U, TU + multiples of 10, TU + TU, U + U + U, TU - U, TU - tens, TU - TU, U x U, U ÷ U and TU ÷ U (within 2,5 and 10x tables)

Year 3: add numbers with up to three digits, HTU + multiples of 10, HTU + multiples of 100, subtract numbers up to three-digits, HTU - U, HTU - multiples of 10, HTU - multiples of 100, HTU - HTU, TU x U, TU ÷ U

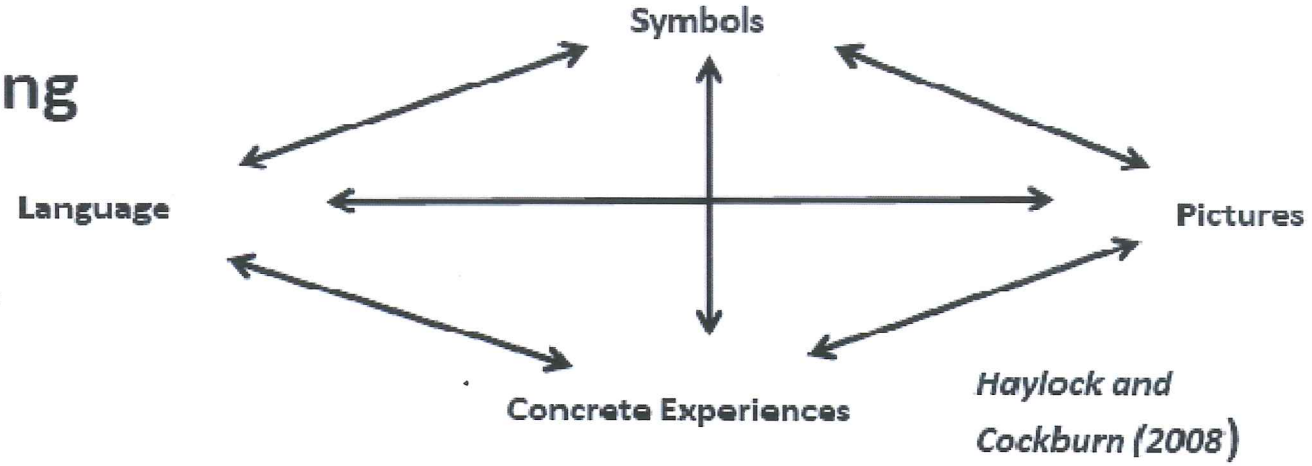
Year 4: add and subtract numbers with up to four-digits, ThHTU + ThHTU, ThHTU - ThHTU, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, TU x U, HTU x U, multiply by zero and one, TU ÷ U, HTU ÷ U

Year 5: add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, ThHTU x U, ThHTU x TU, HTU x TU, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to context)

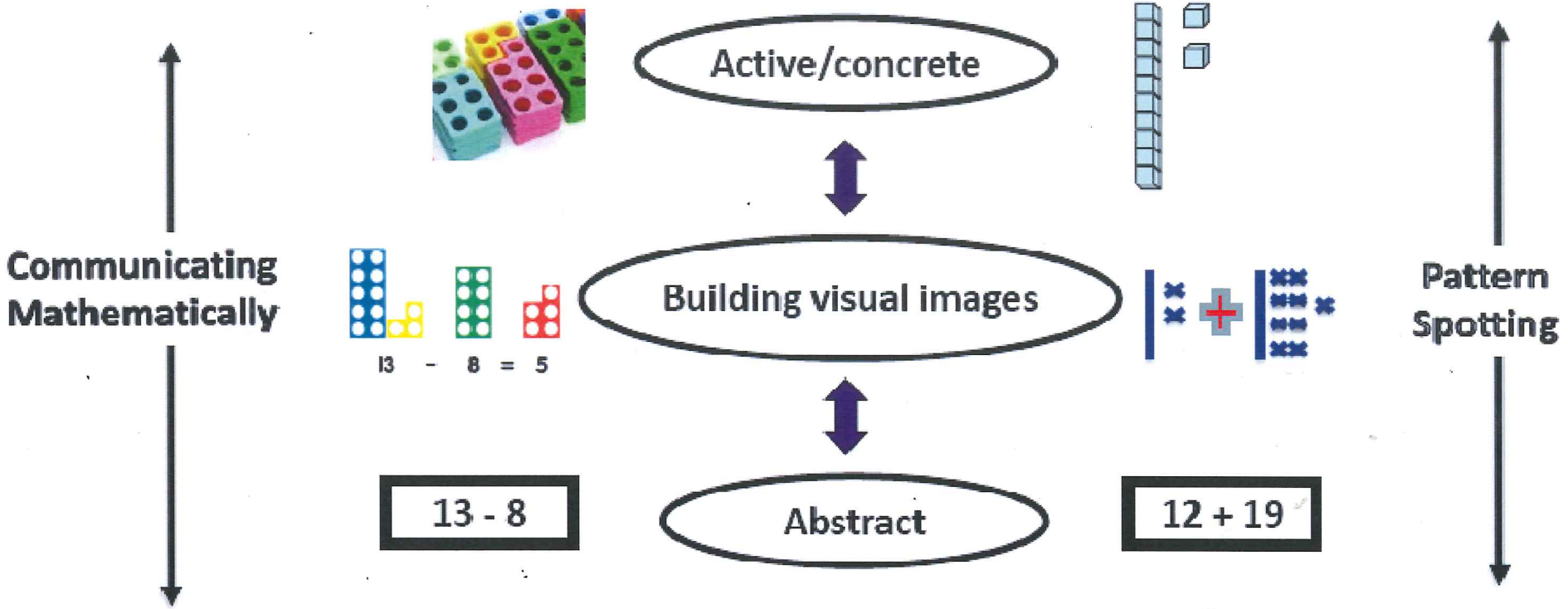
Year 6: add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU, multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.

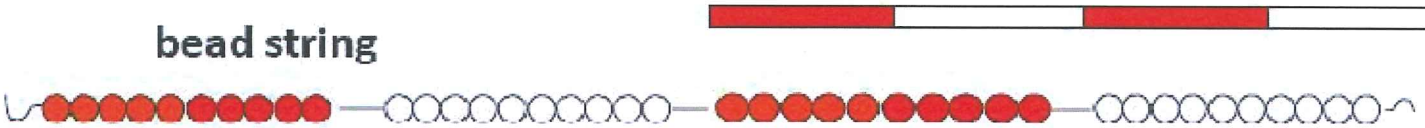


Haylock and Cockburn (2008)

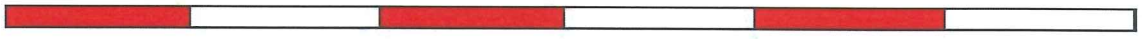


Classroom resources to aid calculation

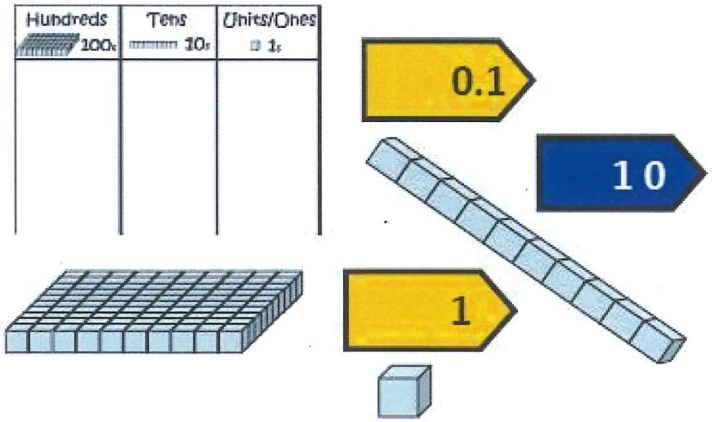
bead string



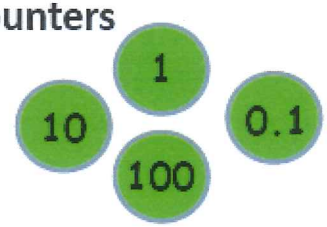
count stick



place value apparatus

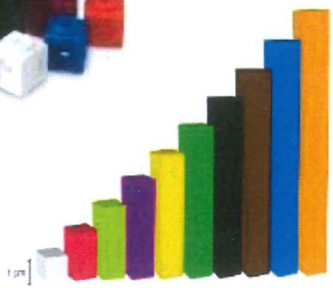


place value counters

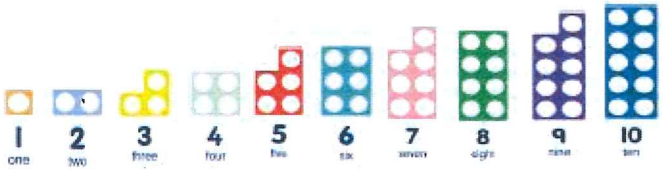


cubes

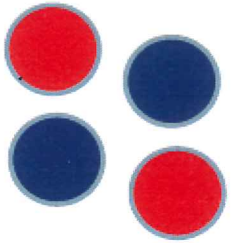
Cuisenaire



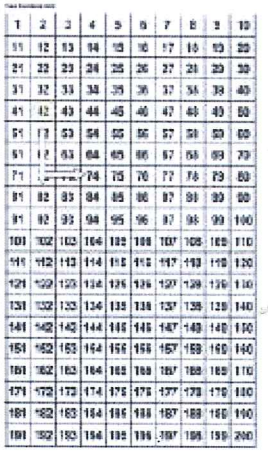
Numicon

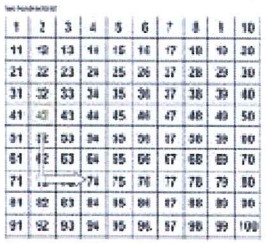


double sided counters

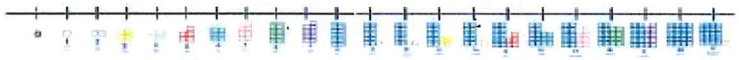


number grids 100 and 200





number line

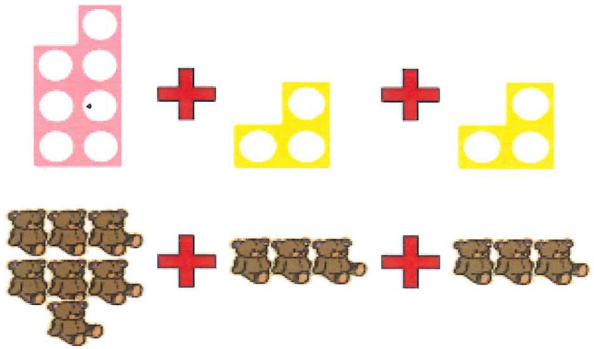


Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

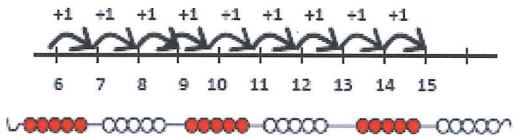
Aggregation

Union of two sets
How many/much altogether?
The total



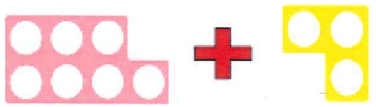
Augmentation

Start at and count on
Increase by
Go up by

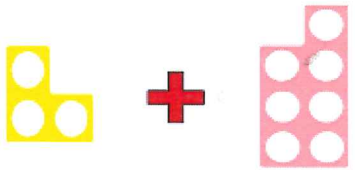


Commutative law

Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)



is the same as/equal to (=)



Addition Vocabulary

Reception

add
count on and
more total
altogether sum

Year 1

and plus
add count on
more total
altogether sum

Year 2

count on
add addition
plus and
more total
altogether sum

Year 3

and plus
count on
add addition
more total
altogether sum

Year 4

count on and
plus
add addition
more sum total
altogether increase

Year 5

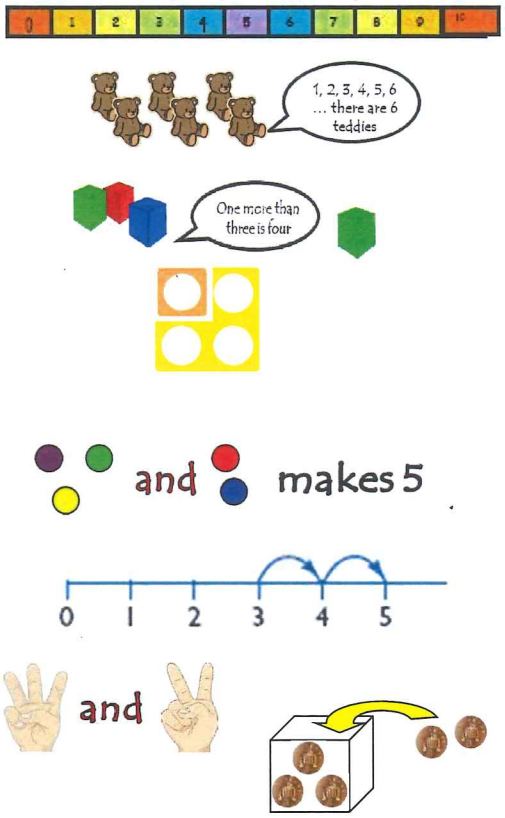
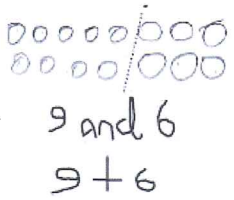
and count on
plus
add addition
more sum total
altogether increase

Year 6

count on
add and
addition plus
more sum total
altogether increase

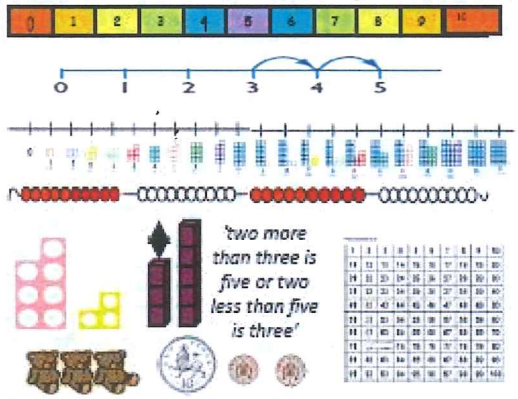
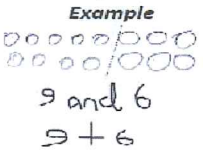
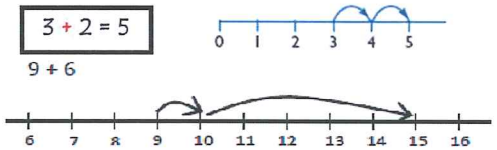
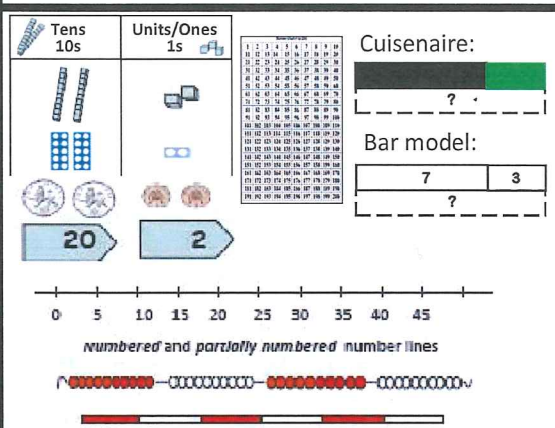
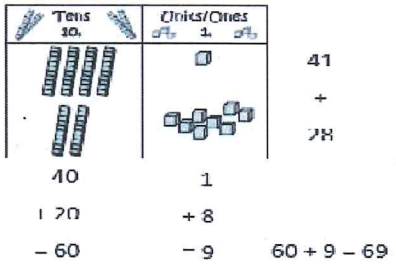
Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations (Early Learning Goals)	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Reception *Magnitude of numbers: U + U* Can say the number that is 1 more than any given number Using quantities and objects they add two single digit numbers and count on	Rhymes and stories *e.g. 1,2,3,4,5, once I caught.... 1,2, buckle my shoe*  **Use practical resources such as bears, cars, counters, cubes and number lines and progress to a resource such as Numicon**	**Always done practically using Numicon or non-structured resources** Children may record using their own images, making marks that they can explain Extend to recording number sentences alongside the practical by the end of the year for those who are ready Use of drawings initially, progressing to digits when ready Children may record pictorially progressing to recording number sentences alongside  **Evidence to include observations, photographs, notes from discussions**	Recognise and order numbers 0 to 20 Count reliably up to 20 everyday objects Find one more than a given number up to 20 Counting on in 1s from 1 to 20, progressing to counting on in 1s from any given number up to 20

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Year 1 *Magnitude of numbers: U + U, U + TU (numbers up to 10) including adding zero* Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus Children must experience increasing numbers e.g. what is two more than seven ? Compare quantities to say how many less and/or how many more	 **Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones**	If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside  Children may record pictorially progressing to recording number sentences alongside  **Jumping in 1s first then progressing to jumps of different sizes. Start from the bigger number** 	Count forwards, to and across 100, beginning with 0 or 1 or from any given number Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ... Represent and use number bonds up to 20 (establish addition and subtraction as related operations) Find one more than a number Find ten more than a number Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition
Year 2 *Magnitude of numbers: TU + U, TU + multiples of 10, TU + TU, U + U + U* Children should be able to partition numbers in different ways e.g. as 2+2+2+1 or 5+3 or 23 as 20 +3 or 10+13 Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS Children should understand the language of sum . **Ensure children understand that addition is commutative (can be done in any order), e.g. starting from the bigger number.**	 **Use practical resources such as bears, counters, cubes and Numicon, number grids, a range of number lines, place value apparatus/Dienes, place value grids, place value cards. Encourage children to partition numbers rather than counting in ones.**	Children apply, develop and secure their understanding of place value, e.g. 17 = 1 ten and 7 ones (7 units) **Continue use of empty number lines to record as appropriate with efficient jumps, e.g. 10s** Use jottings and record number sentences with a pictorial or concrete image to represent E.g. 	Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20 Use knowledge to derive and use number facts up to 100 Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 3

Magnitude of numbers: numbers with up to 3 digits, HTU + multiples of 10, HTU + multiples of 100

Add numbers with up to three-digits

(leading to formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS WITH DIFFERING NUMBERS OF DIGITS

Children should partition numbers, up to 1000, in different ways

e.g. 146 = 100 + 40 + 6 or 100 + 30 + 16

Solve problems in different contexts including missing number problems

Year 4

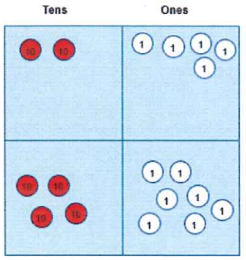
Magnitude of numbers: numbers with up to four digits, ThHTU + ThHTU, decimals with up to two decimal places in the context of money

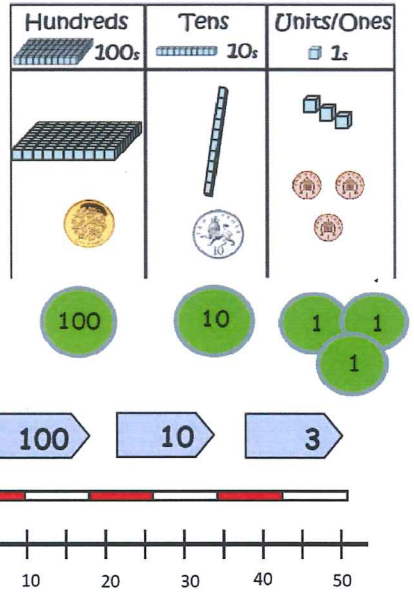
Add numbers with up to four-digits **(formal written column method)** including numbers with up to two decimal places in the context of money

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

Possible Concrete and Visual Representations

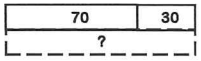




Partially numbered **and** blank number lines

Cuisenaire:

Bar model:



Teacher Modelling and Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns .

Manipulative resources such as place value apparatus SHOULD be used alongside calculations

Expanded column addition (without exchange initially) with up to three-digits:

T	U
40	+ 1
+ 20	+ 8
60	+ 9 = 69

Expanded recording without exchange

T	U
40	+ 3
20	+ 8
70	+ 1 = 71
10	

Expanded recording with exchange

H	T	U
100	+ 40	+ 1
+ 100	+ 20	+ 8
200	+ 60	+ 9 = 269

Expanded recording

Progressing to compact when ready:

H	T	U
	1	4 1
+	1	2 8
	2	6 9

Compact (column) recording

H	T	U
	1	4 3
+	1	2 8
	2	7 1
	1	

Column addition (with exchanging)

H	T	U
	7	8 9
+	6	4 2
1	4	3 1
1	1	

Add decimals in the context of money

£		7.8 9
+	£	6.4 2
£	1	4.3 1
	1	1

Fluency

(Recall & application of number facts)

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Magnitude of numbers: numbers with more than four digits, decimals with up to three decimal places

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Perform mental calculations with increasingly large numbers

Year 6

Magnitude of numbers: numbers with more than four digits, decimals with up to three decimal places

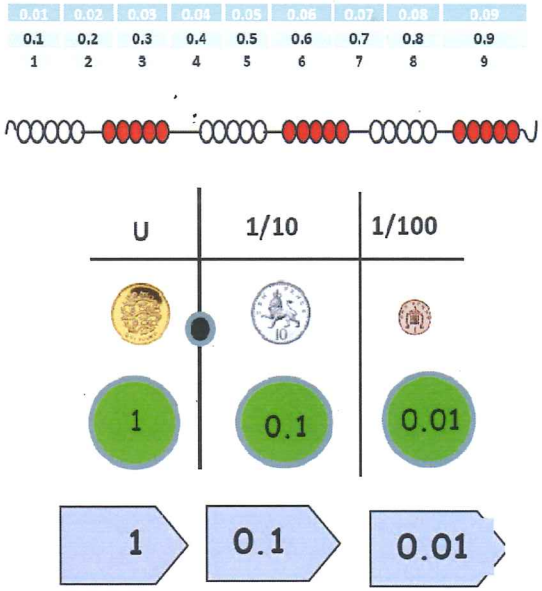
Add numbers with more than four-digits and decimals up to three places

(formal written column method)

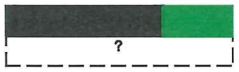
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN** TWO NUMBERS, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve more complex calculations mentally

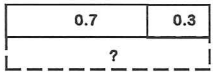
Possible Concrete and Visual Representations

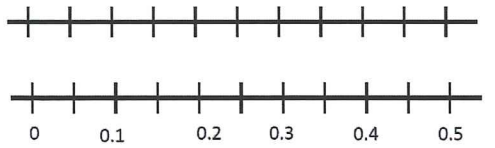


Cuisenaire:



Bar model:





Partially numbered and blank number lines

Teacher Modelling and Children's Recording

Manipulative resources such as base 10 apparatus or place value counters could be used alongside algorithms

```
  2141
+ 1128
------
  3269
```

```
  21.41
   1.12
+  0.35
-------
  22.88
```

Column addition (no exchanging)

```
  5189
+ 3128
------
  8317
  11
```

```
  51.89
+  3.128
--------
  55.018
  1 1
```

Column addition (with exchanging)

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

(Recall & application of number facts)

Count forwards in powers of ten up to 1 000 000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

Count in tens and hundreds increasing fluency of order and place value

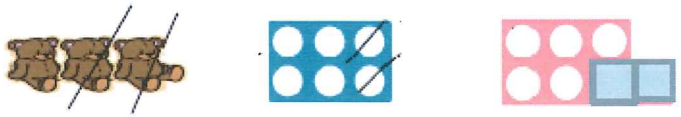
Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

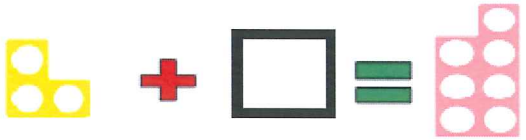
Partitioning

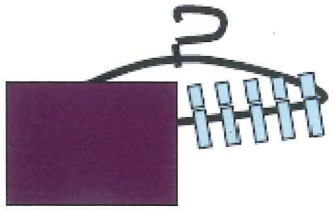
Take away
... how many left?
How many are not?
How many do not?



Inverse-of-addition

What must be added?
How many (much) more needed?

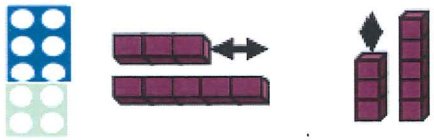




There are ten pegs on the hanger – how many are covered?

Comparison

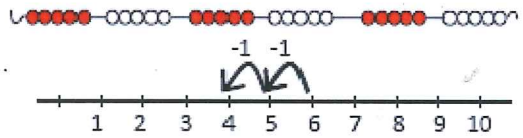
What is the difference?
How many more?
How many less (fewer)?
How much greater?
How much smaller?



'two more than three is five or two less than five is three'

Reduction

Start at and reduce by
Count back by
Go down by



Subtraction Vocabulary

Reception

take away
leave
fewer
count back
difference between

Year 1

take away
leave
subtract
fewer
minus
count back
difference between

Year 2

take away
leave
fewer
subtract
minus
count back
difference between

Year 3

take away
leave
subtract
fewer
minus
count back
difference between

Year 4

take away
leave
subtract
fewer
minus
count back
difference between

Year 5

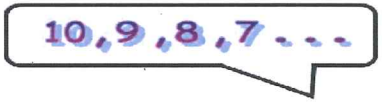
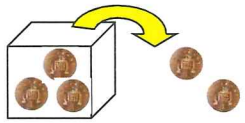
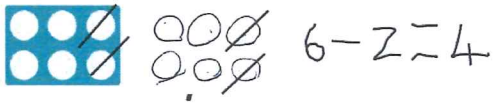
take away
leave
subtract
fewer
minus
count back
difference between

Year 6

take away
leave
subtract
fewer
minus
count back
difference between

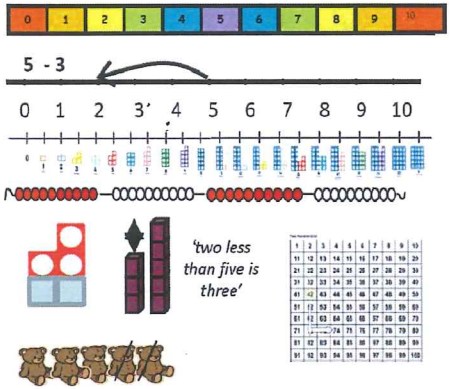
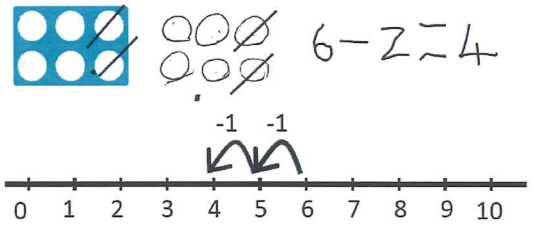
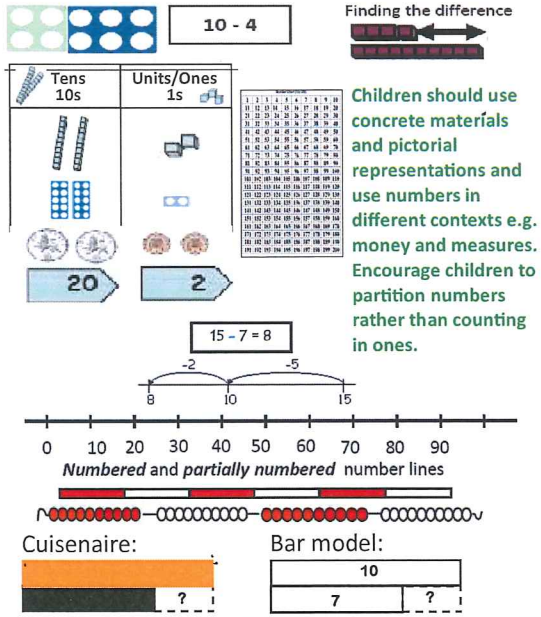
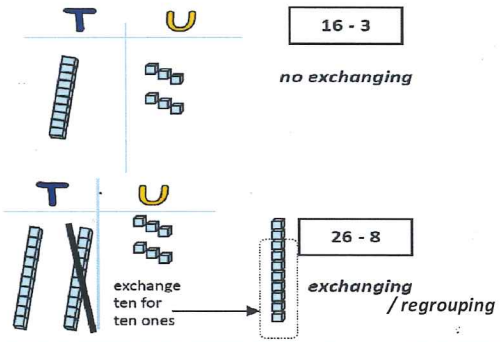
Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations (Early Learning Goals)	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Reception *Magnitude of numbers: U - U* Can say the number that is 1 less than any given number Using quantities and objects they subtract two single digit numbers and count back	**Rhymes and stories**        **Use practical resources such as bears, cars, counters, cubes and number lines and progress to a resource such as Numicon**	**Always done practically using Numicon or non-structured resources** Children may record using their own images, making marks that they can explain Use of drawings initially, progressing to recording number sentences alongside the practical by the end of the year for those who are ready  **Evidence to include observations, photographs, notes from discussions**	Recognise and order numbers up to 20 (0-20, 20-0) Find 1 less than a given number up to 20. Counting back in 1s from 10, then 20, progressing to counting back in 1s from any given number.

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Year 1 *Magnitude of numbers: U - U, TU - U (numbers up to 20) including subtracting zero* Understand subtraction as taking away (Partitioning structure) What is ... less than ...?) Compare quantities to say how many less and/or how many more	 **Use practical resources such as bears, counters, cubes, number lines/hundred grids and Numicon to encourage counting back in groups rather than ones**	**Children may begin recording pictorially as well as using concrete equipment, progressing to recording number sentences alongside**  *Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside*	Count backwards (including crossing 100) from any given number Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10 Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations) Find one less than a number Find ten less than a number Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern
Year 2 *Magnitude of numbers: TU - U, TU - tens, TU - TU* Understand subtraction as taking away (partitioning structure) **and** finding the difference (comparison structure) Ensure children understand that subtraction is not commutative (can not be done in any order) Children should be able to partition numbers in different ways	 **Children should use concrete materials and pictorial representations and use numbers in different contexts e.g. money and measures. Encourage children to partition numbers rather than counting in ones.**	**Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences (alongside the concrete representation)** **Continue use of empty number lines to record as appropriate with efficient jumps, e.g. 10s** 	Practise addition and subtraction facts to 20 Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20 Use known facts to 20 to derive new facts e.g. 10 - 7=3 so 100 - 70=30 Use knowledge to derive and use subtraction number facts up to 100 Mentally calculate TU - U, TU - TU (two digits - tens)

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 3

Magnitude of numbers: numbers up to three digits, HTU - U, HTU - multiples of 10, HTU - multiples of 100, HTU - HTU

Subtract numbers with up to three-digits **(formal written column method)**

Children apply, develop and secure their understanding of place value and begin to record in columns

Year 4

Magnitude of numbers: numbers with up to four digits, ThHTU - ThHTU, decimals with up to two decimal places in the context of money

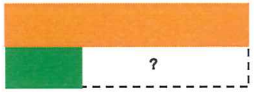
Subtract numbers with up to four-digits **(formal written column method)**

Understand subtraction as the inverse of addition

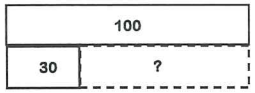
Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made

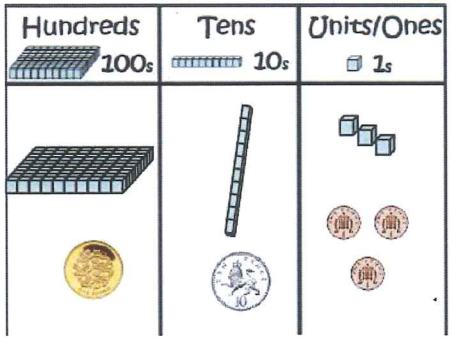
Possible Concrete and Visual Representations

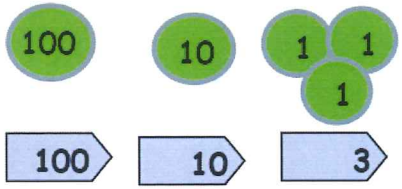
Cuisenaire:

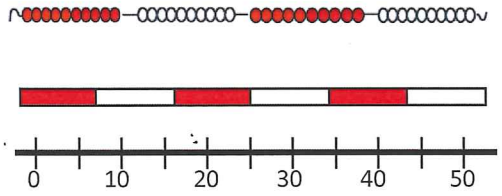


Bar model:









Teacher Modelling and Children's Recording

Children SHOULD use manipulative place value resources alongside calculations to transition between practical and abstract

Expanded column subtraction (without exchange initially) with up to three-digits:

Without exchange:

T U
60 and 8
− 20 and 3
40 + 5 = 45

With exchange:

T U
50 60 and 10 + 3
− 20 and 8
30 + 5 = 35

H T U
100 and 40 and 8
− 100 and 20 and 1
0 + 20 + 7 = 27

Progressing to compact when ready:

HTU
148
− 121
27

H T U
7 1/2 1/3
− 3 1 7
4 0 6

6 11 1
7 /2 3
− 3 6 7
3 5 6

6 11 1
£7 /2 3
− £3 . 6 7
£3 . 5 6

Include calculations where zero is a place holder

Fluency

(Recall & application of number facts)

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice

Switch count between hundreds, tens and ones e.g. 500, 400, 300, 290, 280, 270, 269, 268, 267

Mentally calculate HTU - U, HTU - T, HTU - H

Perform mental calculations with two digit numbers

Find ten and a hundred less than a number with up to three-digits

Count back in 6, 7, 9, 25 and 1000

Count back through zero to include negative numbers

Find 1000 less than a number

Continue to practise mental calculations with increasingly large numbers to aid fluency

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Magnitude of numbers: numbers with more than four digits, decimals with up to three decimal places

Subtract larger numbers **(formal written column method)**

ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Subtract numbers mentally with increasingly large numbers

Year 6

Magnitude of numbers: numbers with more than four digits, decimals with up to three decimal places

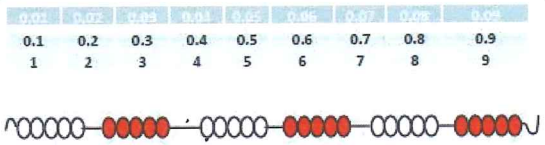
Subtract multi-digit numbers including numbers with up to three decimal places **(formal written column method)**

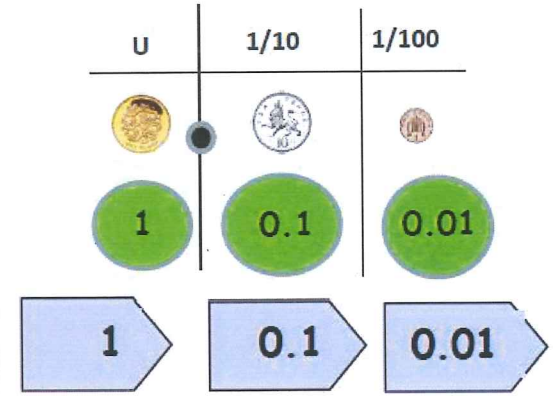
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems in contexts, deciding which operations and methods to use and why

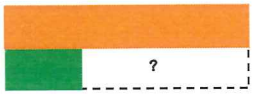
Solve more complex calculations mentally

Possible Concrete and Visual Representations

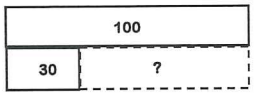


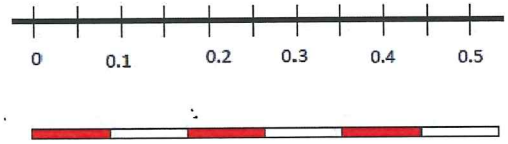


Cuisenaire:



Bar model:





Teacher Modelling and Children's Recording

Children might use manipulatives resources such as base 10 apparatus or place value counters alongside algorithms

Column subtraction (no exchanging/regrouping)

```
 13548
-12128
 ----
  1420
```

Column subtraction (with exchanging/regrouping)

```
 0 1 2 1
 1 2 3 6 5
-   5 1 7 3
 -------
   7 1 9 2
```

```
  2 13 11 1
  1 3 4 2 3
- 1 2 6 7 8
 ---------
     7 4 5
```

Include calculations where zero is a place holder

Column subtraction of decimals (no exchanging)

```
  1.48
- 1.21
 ----
  0.27
```

Column subtraction of decimals (with exchanging)

```
 6 11 1
 7.2 3
-3.6 7
 -----
 3.5 6
```

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

(Recall & application of number facts)

Count backwards in powers of ten from any starting number up to one million

Count backwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

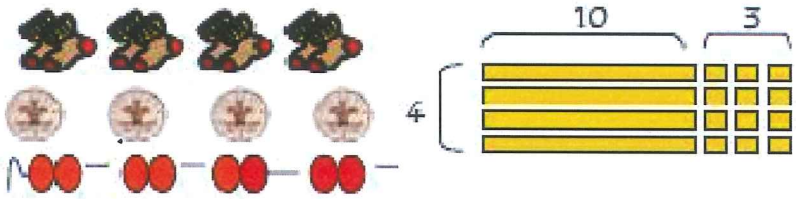
Undertake mental calculations with increasingly large numbers and more complex calculations

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

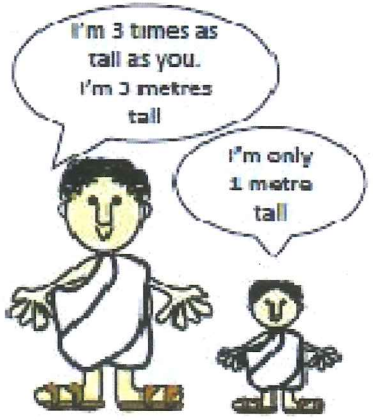
Repeated addition

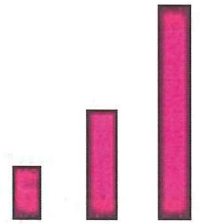
So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)





scaling with Cuisenaire

Commutative law

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal

4 x 2 is the same as/equal to 2 x 4

Multiplication Vocabulary

Reception

double

Year 1

double

Year 2

groups of
lots of
multiplied
multiply
double
times
array
multiplication

Year 3

product
multiplied
lots of
double
groups of
array
multiply
times
multiplication

Year 4

product
multiplied
lots of
double
groups of
array
multiply
times
multiplication

Year 5

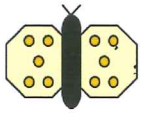
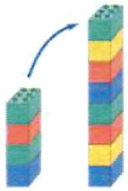
multiplied
product
double
lots of
groups of
array
multiply
times
multiplication

Year 6

lots of
multiplied
product
double
groups of
array
multiply
times
multiplication

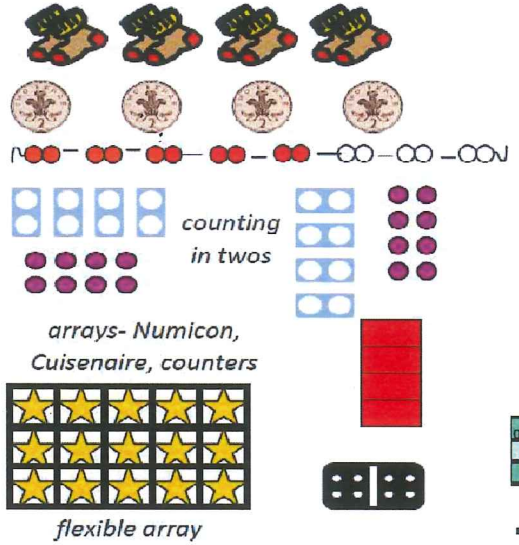
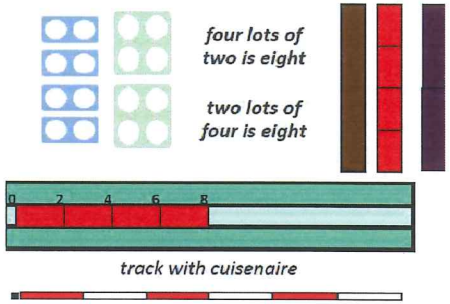
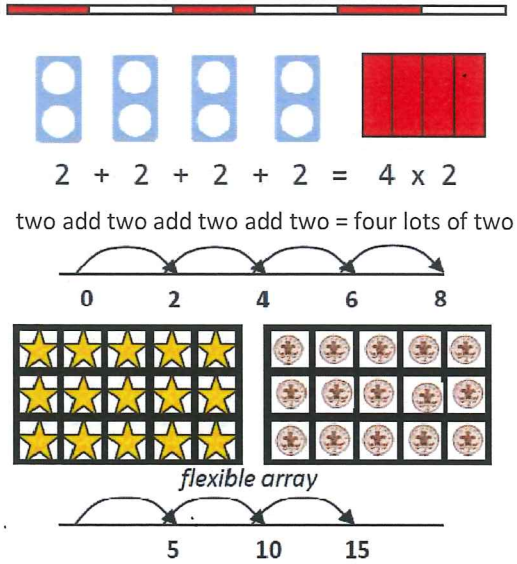
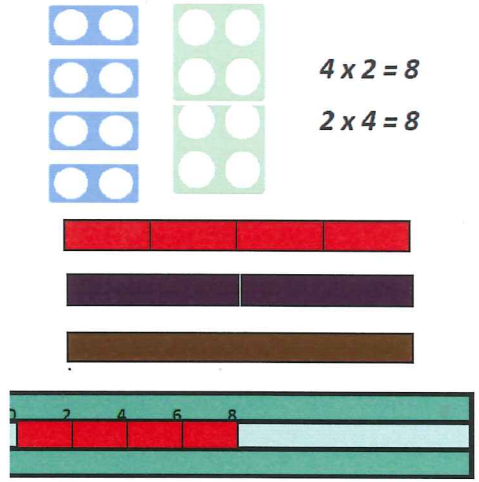
Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations (Early Learning Goals)	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Reception *Magnitude of numbers: doubling single digit numbers* Solve problems involving doubling	 5 + 5 = 10   double 4 is 8 **Use practical resources such as bears, cars, counters, cubes and progress to a resource such as Numicon**	**Always done practically using Numicon or non-structured resources** Children may record using their own images, making marks that they can explain Extend to recording number sentences alongside the practical by the end of the year for those who are ready (repeated addition e.g. 5 + 5 = 10) Use of drawings initially, progressing to digits when ready (repeated addition e.g. 5 + 5 = 10) **Evidence to include observations, photographs, notes from discussions**	Doubling numbers up to 5 *(or further if ready)* *Counting in 2s to 10 when ready*

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Year 1 *Magnitude of numbers: U x U* Solve single step practical problems involving multiplication Use concrete objects, pictorial representations to explore grouping Make connections between arrays, number patterns and counting in twos, fives and tens		**Practical only e.g. link to small world** Using concrete objects, pictorial representations and arrays with the support of an adult - take photographs / draw pictures. If using Numicon, small icons could be stuck in 	Count in twos, fives and tens from different multiples *e.g. 6, 8 , 10, 12 etc* Emphasise number patterns Double number and quantities
Year 2 *Magnitude of numbers: U x U* Understand multiplication as repeated addition Calculate mathematical statements for multiplication within the tables and write them using symbols Understand and solve problems involving arrays Ensure *children understand that multiplication is commutative (can be done in any order)* Understand that multiplication and division are inverse operations		**Record practical work as number sentences** 	Count in twos, threes, fives from zero and tens from any number *e.g. 6, 8 , 10, 12 etc* Emphasise number patterns Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10 Solve multiplication problems mentally

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 3

Magnitude of numbers: TU x U

Develop reliable written methods

Understand and solve scaling problems

Solve problems involving multiplication including correspondence

Year 4

Magnitude of numbers: multiply three numbers together, TU x U, HTU x U, multiply by zero and one

Multiply three numbers

Solve two-step problems

Multiplying by 0 and by 1

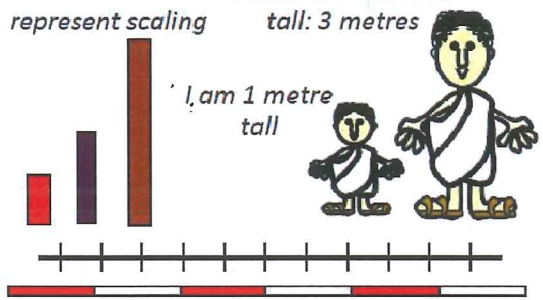
Develop fluency in short multiplication using formal written layout

Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems

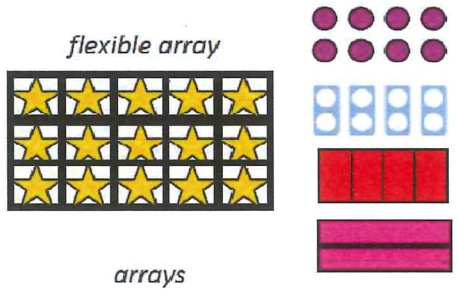
Possible Concrete and Visual Representations

Cuisenaire to represent scaling

Statue is 3 times as tall: 3 metres



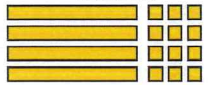
flexible array



arrays

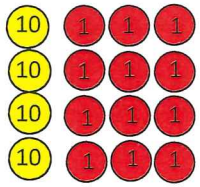
13 x 4

Base 10



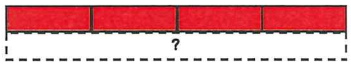
Cuisenaire

place value counters

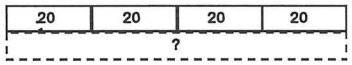


13 x 4

Cuisenaire:



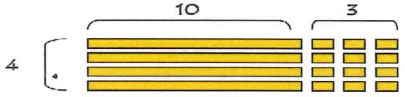
Bar model:

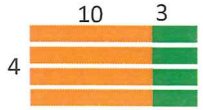


Teacher Modelling and Children's Recording

Children MUST use manipulative resources, e.g. place value apparatus, alongside algorithms.

13 x 4 'thirteen multiplied by four' *or* 'four lots of thirteen'





Expanded methods - grid and area:

	10	3
4	40	12

x	10	3
4	40	12

Progressing to developing fluency in short multiplication:

$$\begin{array}{r} 13 \\ \times\ 4 \\ \hline 52 \\ 1 \end{array} \qquad \begin{array}{r} 133 \\ \times\ 4 \\ \hline 532 \\ 1\ 1 \end{array}$$

Start with digits that are five or below so children can practise the method without encountering difficulty with multiplication tables

Fluency

(Recall & application of number facts)

Count from 0 in multiples of 4, 8 , 50 and 100

Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100

Practise mental recall of multiplication tables: 3, 4 and 8x tables *(also revise 2, 5 and 10x)*

Connect the 2, 4 and 8 times tables using doubling

Develop efficient mental methods using commutativity and multiplication facts to derive related facts,
e.g. 4 x 5 x 12 = 12 x 4 x 5 = 12 x 20

Count in multiples of 6, 7, 9, 25 and 1000

Recall and use multiplication facts up to 12 x 12 with increasing fluency

Derive multiplication facts with up to three digits

Recognise and use factor pairs and commutativity in mental calculations

Use the distributive law

Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations,
e.g. 2 x 6 x 5 = 10 x 6

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 5

Magnitude of numbers: ThHTU x U, HTU x TU, ThHTU x TU, multiply whole numbers and decimals with up to three-decimal places by 100, 100 and 1000

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

Solve problems involving multiplication and division including scaling by simple fractions

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise square and cube numbers and associated notation

Year 6

Magnitude of numbers: numbers with more than four digits, decimals with up to three decimal places

Multiply numbers up to 4 digits by TU

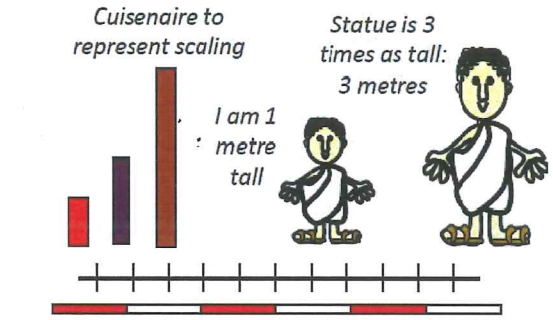
Multiply numbers with up to two decimal places by a whole number

Multiply multi-digit numbers up to four digits by a two digit whole number

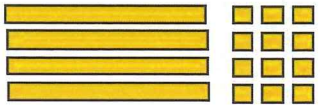
Multiply single digit numbers with up to two decimal places by whole numbers

Solve problems involving all four operations

Possible Concrete and Visual Representations

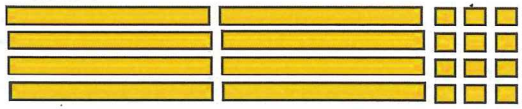


flexible array

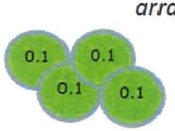


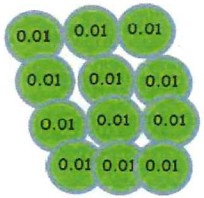
13 x 4

23 x 4



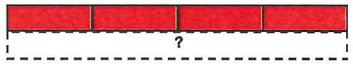
arrays



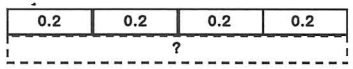


place value counters

Cuisenaire:



Bar model:

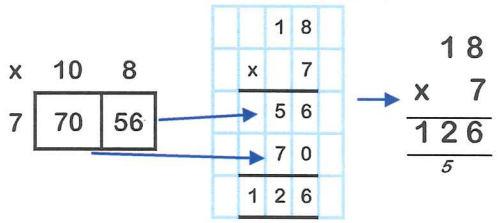


Teacher Modelling and Children's Recording

Children might use manipulative resources, e.g. place value apparatus, alongside algorithms

Progress from grid to expanded columns and finally to the compact (formal) recording, making the link between each step explicit:

.SHORT MULTIPLICATION *(xU):*

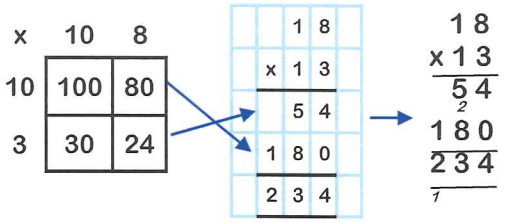


$$\begin{array}{r} 18 \\ \times\ \ 7 \\ \hline 126 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 1324 \\ \times\ \ \ \ 6 \\ \hline 7944 \\ \hline 1\ 1\ 2 \end{array}$$

$$\begin{array}{r} 3.24 \\ \times\ \ \ \ 6 \\ \hline 19.44 \\ \hline 1\ \ 2 \end{array}$$

LONG MULTIPLICATION *(xTU):*



$$\begin{array}{r} 18 \\ \times 13 \\ \hline 54 \\ 180 \\ \hline 234 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1324 \\ \times\ \ \ 26 \\ \hline 7944 \\ 26480 \\ \hline 34424 \\ \hline 1\ 1\ 1 \end{array}$$

$$\begin{array}{r} 3.24 \\ \times\ \ \ 26 \\ \hline 19.44 \\ 64.80 \\ \hline 84.24 \\ \hline 1\ 1 \end{array}$$

Fluency

(Recall & application of number facts)

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of the formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally by drawing upon known facts

Undertake mental calculations with increasingly large numbers

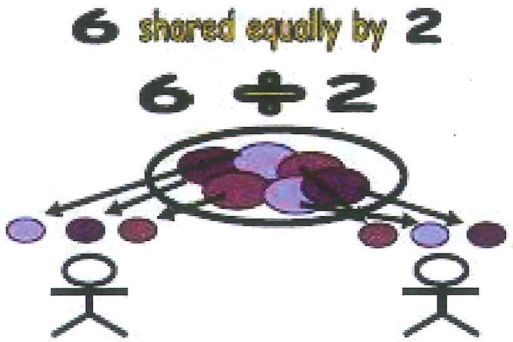
Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Structures for Division (Haylock and Cockburn 2008)

Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

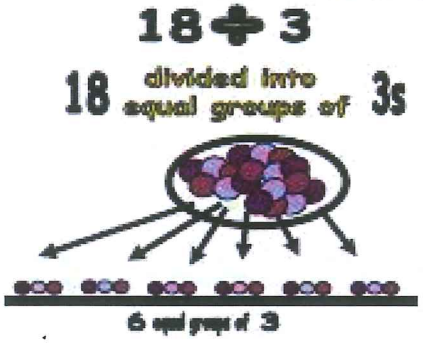
Equal-sharing

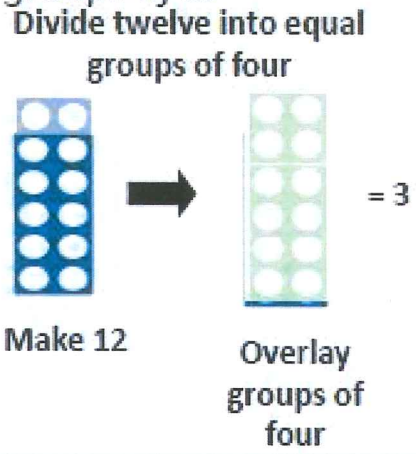
Sharing equally between
How many (much) each?



Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of ...





Ratio structure

comparison
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as long as Ella's. If Jo walks to school in 30 minutes how long does it take Ella?

Division Vocabulary

Reception

halve

half

Year 1

halve

half

Year 2

halve division

divided by share

left over divide

groups of

half lots of

Year 3

halve share

divided by divide

left over lots of

remainder division

half groups of

Year 4

equal groups of

divided by divide

quotient

left over factor

divisible by

remainder share equally

half halve share

Year 5

groups of lots of

divided by divide

quotient

left over factor

divisible by

remainder division

half halve share

Year 6

groups of lots of

divided by divide

quotient

left over factor

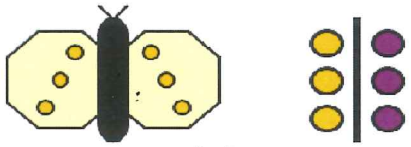
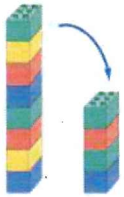
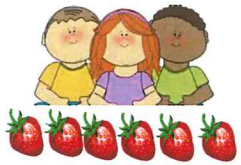
divisible by division

remainder share

half halve

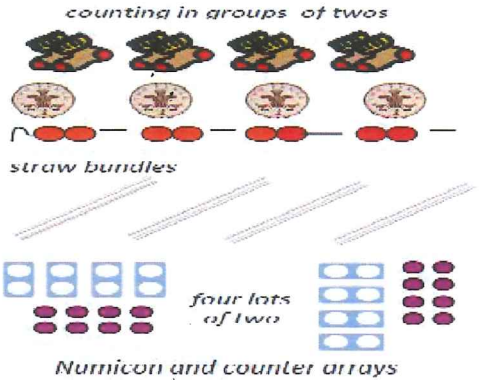
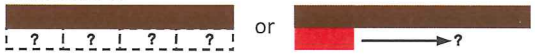
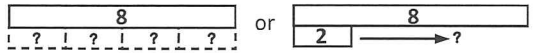
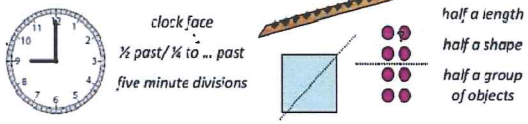
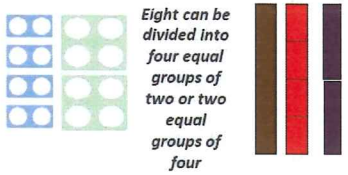
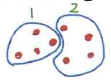
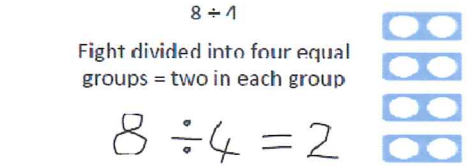
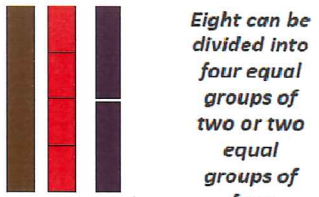
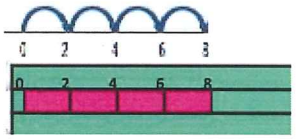
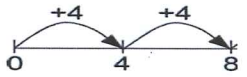
Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations (Early Learning Goals)	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Reception *Magnitude of numbers: Halving and sharing numbers up to 10* Solve problems involving halving and sharing	 Half of 6 is 3  half of 8 is 4 **Equal sharing of small numbers between 2 or more** E.g.  *4 shared between 2 is 2 each .*  *6 shared between 3 is 2 each* **Use practical resources such as bears, cars, counters, cubes, play dough and progress to a resource such as Numicon**	**Always done practically using Numicon or non-structured resources** Children may record using their own images, making marks that they can explain Extend to recording number sentences alongside the practical by the end of the year for those who are ready Use of drawings initially, progressing to digits when ready **Evidence to include observations, photographs, notes from discussions**	Halving even numbers up to 10

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling and Children's Recording	Fluency *(Recall & application of number facts)*
Year 1 *Magnitude of numbers: U ÷ U* Solve single step practical problems involving division Use concrete objects, pictorial representations Understand division as both grouping equally and sharing equally Use the language of 'sharing equally between'	**8 divided into groups of 2:**   doubling/halving Cuisenaire:  Bar model:   	**Practical only** Using concrete objects, e.g. small world, pictorial representations and arrays with the support of an adult - take photographs / draw pictures. If using Numicon, small icons could be stuck in  **Sharing:** **8 shared equally between 4**  **Grouping:** **8 divided into groups of 4** 	Count back in twos, fives and tens from different multiples *e.g. 12, 10, 8, 6 etc* and emphasise number patterns Find simple fractions e.g. half and quarter of objects, numbers and quantities
Year 2 *Magnitude of numbers: U ÷ U* Solve single step practical division problems involving grouping and sharing Use concrete objects, pictorial representations Understand division as grouping and sharing Find halves and then quarters Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays		**Record as number sentences using ÷ and =**    **Empty number line:** 	Count back in twos, threes, fives from different multiples and emphasise number patterns Connect ten times table to place value and five times table to divisions on a clock face Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10 Solve division problems involving grouping and sharing

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 3

Magnitude of numbers: TU ÷ U

Develop a reliable method for division

Solve problems involving missing numbers

Solve problems including those that involve scaling

Recognise, find and name ½ and ¼ of an object, shape or quantity

Understand the link between unit fractions and division e.g. connect $^{1}/_{10}$ to division by 10

Year 4

Magnitude of numbers: TU ÷ U, HTU ÷ U

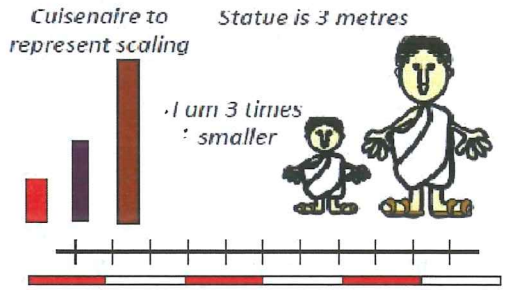
Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number

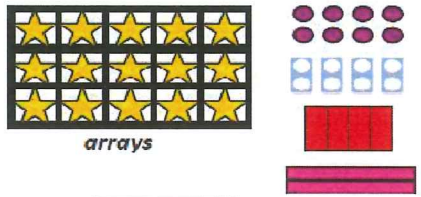
Divide one– or two-digit numbers by 10 or 100, identifying the value of digits as tenths or hundredths

Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers

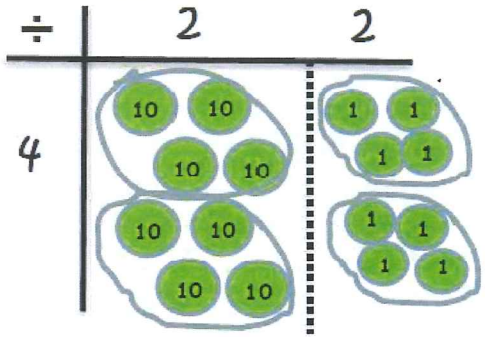
Solve correspondence questions e.g. three cakes shared equally between 10 children

Possible Concrete and Visual Representations



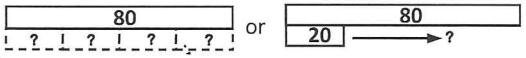


88 ÷ 4



Cuisenaire:

Bar model:



Teacher Modelling and Children's Recording

Children MUST use manipulative resources, such as place value apparatus, alongside algorithms.

Repeated subtraction: chunking

Ensure children see/understand the link between grouping on a number line and vertical recording for chunking

95 ÷ 5 = 19

95
- 50 (10 x 5)
45
- 25 (5 x 5)
20
- 20 (4 x 5)
0

Fact Box

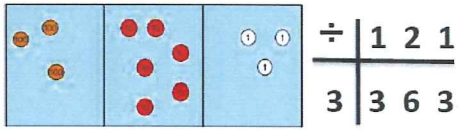
2 x 5 = 10

5 x 5 = 25

10 x 5 = 50

Short division: no remainders

363 ÷ 3



÷	1	2	1
3	3	6	3

560 ÷ 4

÷	1	4	0
4	5	^{1}6	0

Start with problems that involve the multiplication tables that the children are confident with so they can practise the method with familiar facts

Fluency

(Recall & application of number facts)

Recall and use related division facts for the 3, 4 and 8x tables (continue to practise other tables)

Write and calculate mathematical statements for division using known number facts

Use division facts to derive related division facts e.g. using 6 ÷ 3 = 2 to work out 60 ÷ 3 = 20

Count back in tenths

Continue to practise recalling division facts for multiplication tables up to 12 x 12

Practise mental calculations and extend this to three digit numbers for example:
200 x 3 = 600 so 600 ÷ 3 = 200

Use place value, known and derived facts to divide mentally, including dividing by 1

Recognise and use factor pairs to solve division calculations

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.

End of Year Expectations

Year 5

Magnitude of numbers: divide numbers with up to four digits by U (including remainders as fractions and decimals and rounding according to the context)

Identify factors, including finding all factor pairs of a number, and common factors of two numbers

Practise the formal written method of short division with numbers up to four digits by a one-digit number

Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding

Use multiplication and division as inverses

Solve problems involving division including scaling down

Divide whole numbers and those involving decimals by 10, 100 & 1000

Year 6

Magnitude of numbers: divide numbers up to four digits by TU (interpreting remainder according to context), divide decimals up to two decimal places by U or TU

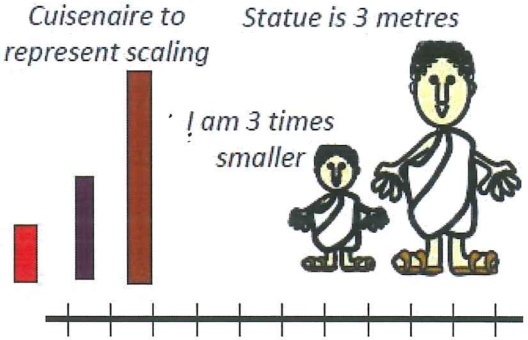
Divide numbers up to 4 digits by a 2 digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context

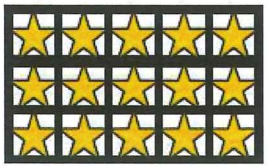
Divide numbers with up to 2 decimal places by 1 digit and 2 digit whole numbers, initially in practical contexts involving money and measures

Understand the relationship between unit fractions and division

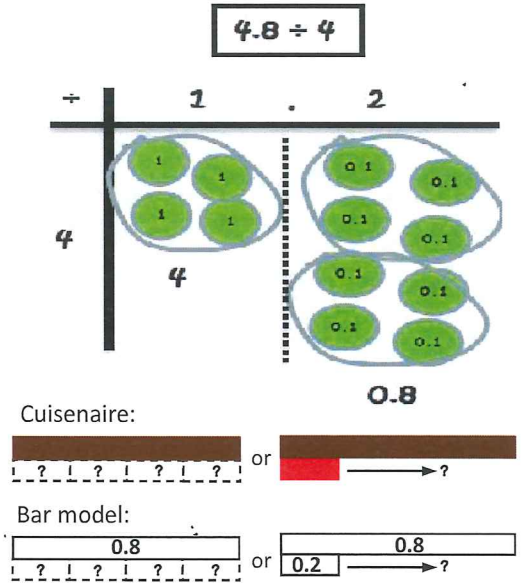
Recognise division calculations as the inverse of multiplication in order to solve ratio problems

Possible Concrete and Visual Representations





flexible arrays



Teacher Modelling and Children's Recording

Children could use manipulative resources, such as place value apparatus, alongside algorithms

SHORT DIVISION:

No remainder:

560 ÷ 4

$$4\overline{)5^{1}60} = 140$$

Remainder as a whole number:

564 ÷ 5

$$5\overline{)5\,6^{1}4} = 112\text{ r }4$$

Remainder as a fraction:

$$5\overline{)5\,6^{1}4} = 112\ \tfrac{4}{5}$$

Remainder as a decimal:

$$5\overline{)5\,6^{1}4.^{4}0} = 112.8$$

LONG DIVISION:

Remainder as a whole

$$25\overline{)535}: 535 - 50 = 35;\ 35 - 25 = 10 \Rightarrow 21\text{ r }10$$

Remainder as a fraction in its lowest form:

$$25\overline{)535}: 535 - 50 = 35;\ 35 - 25 = 10 \Rightarrow 21\text{ r }\tfrac{10}{25}\ \left(\tfrac{2}{5}\right)$$

Remainder as a decimal:

$$25\overline{)535.0}: 535 - 50 = 35;\ 350 - 250 = 100;\ 100 - 100 = 0 \Rightarrow 21.4$$

Fluency

(Recall & application of number facts)

Count backwards in steps of powers of 10 for any given number up to 1 000 000

Count backwards with positive/ negative whole numbers through zero

Practise mental calculation with increasingly large numbers

Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently make larger calculations

Practise using written methods for short and long division

Continue to use all multiplication tables and division facts to maintain fluency

Perform mental calculations including with mixed operations and larger numbers